

# Viability of the Matter Bounce Scenario

Jaume de Haro<sup>\*</sup> and Jaume Amorós<sup>†</sup>

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Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya,  
Diagonal 647, 08028 Barcelona, Spain

## Abstract

It is shown that teleparallel  $F(\mathcal{T})$  theories of gravity combined with Loop Quantum Cosmology support a Matter Bounce Scenario which is an alternative to the inflation scenario in the Big Bang paradigm. It is checked that these bouncing models provide theoretical data that fits well with the current observational data, allowing the viability of the Matter Bounce Scenario.

## 1 Introduction

It is well-known that inflation suffers from several problems (see [1] for a review about these problems), like the initial singularity which is usually not addressed, or the fine-tuning of the degree of flatness required for the potential in order to achieve successful inflation [2].

In order to avoid these problems, an alternative scenario to the inflationary paradigm, called *Matter Bounce Scenario* (MBS), has been developed in order to explain the evolution of our Universe (see [3]). Essentially, it depicts at very early times a matter dominated Universe in a contracting phase, that evolves towards the bounce and afterwards enters an expanding phase. This model, like inflation, solves the horizon problem that appears in General Relativity (GR) and improves the flatness problem in GR (where spatial flatness is an unstable fixed point and fine tuning of initial conditions is required), because the contribution of the spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding one (see for instance [4]).

The aim of our work is to construct viable bouncing cosmologies where the matter part of the Lagrangian is composed of a single scalar field and, therefore, have to go beyond General Relativity, since GR forbids bounces when one deals with a single field. Hence, theories such as holonomy corrected Loop Quantum Cosmology (LQC) [5], where a big bounce appears owing to the discrete structure

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<sup>\*</sup>E-mail: jaime.haro@upc.edu

<sup>†</sup>E-mail: jaume.amoros@upc.edu

of space-time [6] or teleparallelism [7] must be taken into account. When dealing with these theories, in order to obtain a theoretical value of the spectral index and its running that may fit well with current experimental data, a quasi-matter dominated regime in the contracting phase termed by the condition  $\left|w \equiv \frac{P}{\rho}\right| \ll 1$ , where  $P$  and  $\rho$  are respectively the pressure and the energy density of the Universe, has to be introduced [8].

Since in Matter Bounce Scenario the number of e-folds before the end of the quasi-matter domination regime can be relatively small, the horizon problem does not exist in bouncing cosmologies and the flatness problem is neutralized [4]. This argues for the viability of such models, making it possible that for certain matter bounce scenarios the forecast values of the spectral index and of the running parameter agree well with the most accurate current observations.

In contrast, in slow roll inflation one must consider the running of the spectral index corresponding to  $N$  e-folds before the end of the inflation, which in general, is of the order of  $N^{-2}$ . This value turns out to be very small, when one substitutes for  $N$  the minimum number of e-folds which are needed to solve the horizon and flatness problem in inflationary cosmology (the usual accepted value is  $N > 50$ ), as compared with its corresponding observational value  $-0.0134 \pm 0.009$  coming from the most recent Planck data [9]. This shows that these slow roll models are less favored by observations.

The units used in the paper are:  $\hbar = c = 8\pi G = 1$ .

## 2 $F(\mathcal{T})$ gravity in flat FLRW geometry

Teleparallel theories are based in the *Weitzenböck space-time*. This space is  $\mathbb{R}^4$ , with a Lorentz metric, in which a global, orthonormal basis of its tangent bundle given by four vector fields  $\{e_i\}$  has been selected, that is, they satisfy  $g(e_i, e_j) = \eta_{ij}$  with  $\eta = \text{diag}(-1, 1, 1, 1)$ . The Weitzenböck connection  $\nabla$  is defined by imposing that the basis vectors  $e_i$  be absolutely parallel, i.e. that  $\nabla e_i = 0$ .

The Weitzenböck connection is compatible with the metric  $g$ , and it has zero curvature because of the global parallel transport defined by the basis  $\{e_i\}$ . The information of the Weitzenböck connection is carried by its torsion, and its basic invariant is the *scalar torsion*  $\mathcal{T}$ . The connection, and its torsion, depend on the choice of orthonormal basis  $\{e_i\}$ , but if one adopts the flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric and selects as orthonormal basis  $\{e_0 = \partial_0, e_1 = \frac{1}{a}\partial_1, e_2 = \frac{1}{a}\partial_2, e_3 = \frac{1}{a}\partial_3\}$ , then the scalar torsion is

$$\mathcal{T} = -6H^2, \quad (1)$$

where  $H = \frac{\dot{a}}{a}$  is the Hubble parameter, and this identity is invariant with respect to local Lorentz transformations that only depend on the time, i.e. of the form  $\tilde{e}_i = \Lambda_i^k(t)e_k$  (see [10, 11]).

With the above choice of orthonormal fields, the Lagrangian of the  $F(\mathcal{T})$  theory of gravity is

$$\mathcal{L}_{\mathcal{T}} = \mathcal{V}(F(\mathcal{T}) + \mathcal{L}_M), \quad (2)$$

where  $\mathcal{V} = a^3$  is the volume of the Universe, and  $\mathcal{L}_M$  is the matter Lagrangian density.

The Hamiltonian of the system is

$$\mathcal{H}_{\mathcal{T}} = \left( 2\mathcal{T} \frac{dF(\mathcal{T})}{d\mathcal{T}} - F(\mathcal{T}) + \rho \right) \mathcal{V}, \quad (3)$$

where  $\rho$  is the energy density. Imposing the Hamiltonian constrain  $\mathcal{H}_{\mathcal{T}} = 0$  leads to the modified Friedmann equation

$$\rho = -2 \frac{dF(\mathcal{T})}{d\mathcal{T}} \mathcal{T} + F(\mathcal{T}) \equiv G(\mathcal{T}) \quad (4)$$

which, as  $\mathcal{T} = -6H^2$ , defines a curve in the plane  $(H, \rho)$ .

Equation (4) may be inverted, so a curve of the form  $\rho = G(\mathcal{T})$  defines an  $F(\mathcal{T})$  theory with

$$F(\mathcal{T}) = -\frac{\sqrt{-\mathcal{T}}}{2} \int \frac{G(\mathcal{T})}{\mathcal{T} \sqrt{-\mathcal{T}}} d\mathcal{T}. \quad (5)$$

To produce a cyclically evolving Universe, let us take the  $F(\mathcal{T})$  theory arising from the ellipse that defines the holonomy corrected Friedmann equation in Loop Quantum Cosmology

$$H^2 = \frac{\rho}{3} \left( 1 - \frac{\rho}{\rho_c} \right), \quad (6)$$

where  $\rho_c$  is the so-called *critical density*.

To obtain a parametrization of the form  $\rho = G(\mathcal{T})$ , the curve has to be split in two branches

$$\rho = G_{\pm}(\mathcal{T}) = \frac{\rho_c}{2} \left( 1 \pm \sqrt{1 + \frac{2\mathcal{T}}{\rho_c}} \right), \quad (7)$$

where the branch  $\rho = G_{-}(\mathcal{T})$  corresponds to  $\dot{H} < 0$  and  $\rho = G_{+}(\mathcal{T})$  is the branch with  $\dot{H} > 0$ . Applying Eq. (5) to these branches produces the model ([12, 13, 14])

$$F_{\pm}(\mathcal{T}) = \pm \sqrt{-\frac{\mathcal{T}\rho_c}{2}} \arcsin \left( \sqrt{-\frac{2\mathcal{T}}{\rho_c}} \right) + G_{\pm}(\mathcal{T}). \quad (8)$$

### 3 Matter Bounce Scenario

Matter Bounce Scenarios (see [3] for a recent review) are essentially characterized by the Universe being nearly matter dominated at very early times in the contracting phase (to obtain an approximately scale invariant power spectrum) and evolving towards a bounce where all the parts of the Universe become in causal contact [12], solving the horizon problem, to enter into an expanding regime, where it matches the behavior of the standard hot Friedmann Universe. They constitute an alternative to the inflationary paradigm.

According to the current observational data, in order to obtain a viable MBS model, the bouncing model has to satisfy some conditions that we have summarized as follows:

1. The latest Planck data constrain the value of the spectral index for scalar perturbations and its running, namely  $n_s$  and  $\alpha_s$ , to  $0.9603 \pm 0.0073$  and  $-0.0134 \pm 0.009$  respectively [9]. The analysis of these parameters provided by Planck makes no slow roll approximation (in fact, the determination of cosmological parameters from the first year WMAP observations was done considering the  $\Lambda$ CDM model [15]), which means that the parameters  $n_s$  and  $\alpha_s$  could be used to test bouncing models. On the other hand, it is well-known that the ways to obtain a nearly scale invariant power spectrum of perturbations with running are either a quasi de Sitter phase in the expanding phase or a nearly matter domination phase at early times, in the contracting phase [16]. Then, since for the MBS one has  $n_s = 1$ , if one wants to improve the model to match correctly with this observational data, one has to consider, at early times in the contracting phase, a quasi-matter domination period characterized by the condition  $\left|w \equiv \frac{P}{\rho}\right| \ll 1$ , being  $P$  and  $\rho$  the pressure and the energy density of the Universe.
2. The Universe has to reheat creating light particles that will thermalize matching with a hot Friedmann Universe. Reheating could be produced due to the gravitational particle creation in an expanding Universe [17]. In this case, an abrupt phase transition (a non adiabatic transition) is needed in order to obtain sufficient particle creation that thermalizes producing a reheating temperature that fits well with current observations. This method was used in the context of inflation in [18, 19], where a sudden phase transition from a quasi de Sitter phase to a radiation domination or a quintessence phase was assumed in the expanding regime. It is shown in [20] that gravitational particle production could be applied to the MBS, assuming a phase transition from the matter domination to an ekpyrotic phase in the contracting regime, and obtaining a reheating temperature compatible with current data.
3. Studies of distant type Ia supernovae ([21] and others) provide strong evidence that our Universe is expanding in an accelerating way. A viable model

must take into account this current acceleration, which could be incorporated, in the simplest case, with a cosmological constant, or by quintessence models [22]. There are other ways to implement the current cosmic acceleration, for example using  $F(\mathcal{R})$  gravity (see for instance [23]), but the current models that provide this behavior are very complicated, and the main objective in MBS is to present the simplest viable models.

4. The data of the seven-year survey WMAP ([24]) constrains the value of the power spectrum for scalar perturbations to be  $\mathcal{P}_S(k) \cong 2 \times 10^{-9}$ . The numerical results (analytical ones will be impossible to obtain) calculated with bouncing models have to match with that experimental data.
5. The constrain of the tensor/scalar ratio provided by WMAP and Planck projects ( $r \leq 0.11$ ) is obtained indirectly assuming the *consistency* slow roll relation  $r = 16\epsilon$  (where  $\epsilon = -\frac{\dot{H}}{H^2} \cong \frac{1}{2} \left( \frac{V_\phi}{V} \right)^2$  is the main slow roll parameter) [25], because gravitational waves are not longer detected by those projects. This means that, the slow roll inflationary models must satisfy this constrain, but not the bouncing ones, where there is not any consistency relation. This point is very important because some very complicated mechanisms are sometimes implemented to the MBS in order to enhance the power spectrum of scalar perturbation to achieve the observational bound provided by Planck [26]. In fact, in matter bounce scenario, to check if the models provide a viable value of the tensor/scalar ratio, first of all gravitational waves must be clearly detected in order to determine the observed value of this ratio. The authors hope that more accurate unified Planck-BICEP2 data (the B2P collaboration), which is going to be issued soon, may adress this point. In contrast, as we have pointed out in (i), the spectral index of scalar perturbations and its running could be calculated independently of the theory, which means that in order to check bouncing models, while in the absence of evidence of gravitational waves, one has to work in the space  $(n_s, \alpha_s)$ .

## 4 Perturbations in Matter Bounce Scenario

The Mukhanov-Sasaki equations (see [27] for a deduction of these equations in GR) for  $F(\mathcal{T})$  gravity and LQC are given by [28, 29]

$$\zeta_{S(T)}'' - c_s^2 \nabla^2 \zeta_{S(T)} + \frac{Z_{S(T)}'}{Z_{S(T)}} \zeta_{S(T)}' = 0, \quad (9)$$

where  $\zeta_S$  and  $\zeta_T$  denote the amplitude for scalar and tensor perturbations.

In  $F(\mathcal{T})$  gravity one has

$$Z_S = \frac{a^2 |\Omega| \dot{\varphi}^2}{c_s^2 H^2}, \quad Z_T = \frac{a^2 c_s^2}{|\Omega|}, \quad c_s^2 = |\Omega| \frac{\arcsin \left( 2 \sqrt{\frac{3}{\rho_c}} H \right)}{2 \sqrt{\frac{3}{\rho_c}} H}, \quad \text{with} \quad \Omega = 1 - \frac{2\rho}{\rho_c}. \quad (10)$$

In contrast, for LQC,

$$Z_S = \frac{a^2 \dot{\phi}^2}{H^2}, \quad Z_T = \frac{a^2}{\Omega}, \quad c_s^2 = \Omega. \quad (11)$$

The power spectrum for scalar perturbations is given by [30]

$$\mathcal{P}_S(k) = \frac{3\rho_c^2}{\rho_{pl}} \left| \int_{-\infty}^{\infty} Z_S^{-1}(\eta) d\eta \right|^2, \quad (12)$$

where, in order to obtain this formula, the scale factor  $a(t) \cong (\frac{4}{3}\rho_c t^2)^{1/3}$  at early times has been used. In the particular case of an exactly matter dominated universe during all the background evolution, i.e., when  $a(t) = (\frac{4}{3}\rho_c t^2 + 1)^{1/3}$  for teleparallel  $F(\mathcal{T})$  gravity one has  $\mathcal{P}_S(k) = \frac{16}{9} \frac{\rho_c}{\rho_{pl}} \mathcal{C}^2$ , [31] where  $\mathcal{C} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \dots = 0.915965\dots$  is the Catalan's constant, and for holonomy corrected LQC  $\mathcal{P}_S(k) = \frac{\pi^2}{9} \frac{\rho_c}{\rho_{pl}}$  [32].

The ratio of tensor to scalar perturbations in MBS is given by

$$r = \frac{8}{3} \left( \frac{\int_{-\infty}^{\infty} Z_T^{-1}(\eta) d\eta}{\int_{-\infty}^{\infty} Z_S^{-1}(\eta) d\eta} \right)^2, \quad (13)$$

where the factor 8 appears due to the two polarizations of the gravitational waves and to the renormalization with respect to a canonical field [33].

The spectral index for scalar perturbations and its running are calculated in [8] given

$$n_s - 1 = 12w, \quad \alpha_s = -48\delta^2, \quad (14)$$

where the parameters  $w$  and  $\delta^2$ , calculated in the quasi-matter domination, as a functions of the potential are

$$w \cong \frac{1}{3} \left( \frac{V_\varphi}{V} \right)^2 - 1, \quad \delta^2 \cong - \left( \frac{V_\varphi}{V} \right)_\varphi. \quad (15)$$

#### 4.1 Comparison with observational data in the plane $(n_s, \alpha_s)$

In slow-roll inflation, for the general models (monomial, natural, hilltop and plateau potentials),  $1 - n_s$  is of the order  $N^{-1}$ , while the running parameter is of order  $N^{-2}$  and, consequently, one has  $\alpha_s \sim (1 - n_s)^2$ , which in most cases is incompatible with Planck and WMAP data, because the observed value of the running is not small enough [34, 35].

Thus, the observation of a large negative running implies that any inflationary phase requires multiple fields or the breakdown of slow roll. Following this second path, in [35] the authors consider the break of the slow-roll approximation for a short while, due to the inclusion of a quickly oscillating term in the potential. As a

consequence, the theoretical value of the running parameter gets larger and could match well with observational data.

In contrast, in MBS the situation is completely different. For example, in [8] dealing with a perfect fluid whose Equation of State (EoS) is parametrized by the number of e-folds before the end of the quasi-matter domination period, namely  $N$ , the authors have shown that the theoretical values of the spectral index of scalar perturbations and its running fit well with their corresponding observational data. To be more precise, for the EoS  $P = \frac{\beta}{(N+1)^\alpha} \rho$ , ( $\alpha > 0$ ,  $\beta < 0$ ) the following relation

$$\alpha_s = \frac{2\alpha}{N+1}(n_s - 1) \quad (16)$$

is obtained, which is perfectly compatible with the experimental data. In fact, for instance, if one takes  $\alpha = 2$  and  $N = 12$  (note that in bouncing cosmologies a large number of e-folds is *not* required, because the horizon problem does not exist, since at the bounce all parts of the Universe are already in causal contact, and also the flatness problem gets improved [4]), one obtains, for  $n_s = 0.9603 \pm 0.0073$ , the following value for the running parameter:  $\alpha_s = -0.0122 \pm 0.0022$ , which is compatible with the Planck data. Effectively, for these values of  $\alpha$  and  $N$  one gets  $n_s - 1 = \frac{12}{13^2} \beta \cong 0.071\beta$ , which is indeed compatible with its observed value, by choosing  $\beta \cong -\frac{1}{2}$ .

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